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Global Positioning System Integer Ambiguity Resolution Without Attitude Knowledge

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In this paper, a new motion-based algorithm for global positioning system integer ambiguity resolution is derived. The algorithm represents the global positioning system sightline vectors in the body frame as the sum of two vectors, one depending on the phase measurements and the other on the unknown integers. The vector containing the integer phases is found using a procedure developed to solve for magnetometer biases. In addition to a batch solution, this paper also provides a sequential estimate, so that a suitable stopping condition can be found during the vehicle motion. The new algorithm has several advantages: it does not require an a priori estimate of the vehicle's attitude; it provides an inherent integrity check using a covariance-type expression; and it can sequentially estimate the ambiguities during the vehicle motion. Its only disadvantage is that it requires at least three noncoplanar baselines. The performance of the new algorithm is tested on a dynamic hardware simulator.

Introduction

THE use of phase difference measurements from global positioning system (GPS) receivers provides a novel approach for three-axis attitude determination and/or estimation. These measurements have been used successfully to determine the attitude of air-based,[1] space-based,[2,3] and sea-based[4] vehicles. Because phase differences are used, the correct number of integer wavelengths between a given pair of antennas must be found. The integer ambiguities can be determined using either instantaneous (motionless) or dynamic (motion-based) techniques. The ambiguities essentially act as integer biases to the phase difference measurements. Once the integer ambiguities are resolved, then the attitude determination problem can be solved.[5]

Instantaneous methods find a solution that minimizes the error residual at a specific time by searching through an exhaustive list of all possible integers and rejecting candidate solutions when the residual becomes too large.[6] Refinements can be made to the solution by restricting the search space with knowledge of a priori information, such as the maximum tilt the baseline should encounter.[7] Instantaneous methods generally rely on solving a set of Diophantine equations.[8] The appeal of these methods is that they provide an instantaneous attitude solution, limited only by computation time, and are well suited to short baselines. However, the minimum residual does not guarantee a correct solution in the presence of noise.[9] In fact, it is possible that instantaneous methods can report a wrong solution as valid. This lack of integrity can cause significant problems if the sensor output is used to control a high bandwidth actuator, such as gas jets on a spacecraft. Another consideration is that instantaneous methods sometime require that the antenna array must be within a defined angle (typically 30 deg) of a reference attitude, which is often true for ground-based applications, but is less likely for space-based applications. All of the aforementioned limitations imply that instantaneous methods, although attractive because of their fast solutions, are not totally acceptable for general purpose applications.

Dynamic techniques for resolving integer ambiguities involve collecting data for a given period of time and performing a batch solution, in which the integer terms remain constant over the collection period. These techniques rely on the fact that a certain amount of motion has occurred during the data collection, either from vehicle body rotation or GPS line-of-sight motion. Their main disadvantage, compared with instantaneous approaches, is that it takes time for the motion to occur, which may be on the order of several minutes. Another consideration is that a potentially significant amount of memory is required for the storage of the batch data collection. But motion-based techniques also have significant advantages over instantaneous methods. Most importantly, motion-based techniques are inherently high-integrity methods because there are numerous checks that can be implemented into the solution before it is accepted. These include the use of statistical checks applied to error residuals, matrix condition number checks, and the use of the closeness of the computed floating-point integers to actual integers as a check. The probability of an erroneous solution being reported as valid can be made as small as desired by appropriately setting the thresholds on these integrity checks. For these reasons, motion-based techniques have been more widely used for onboard applications.

Traditional motion-based techniques of integer ambiguity resolution rely on the fact that either GPS line-of-sight motion or vehicle motion dominates the changes in differential carrier phase measurements. Cohen[9] developed an algorithm, known as quasistatic integer resolution, that can be used when the GPS line-of-sight motion and the vehicle rotation both account approximately evenly for the differential carrier phase measurement changes. This algorithm can be adapted to almost any vehicle motion, slow or fast, simply by varying the sample rate and the data collection time. The quasistatic method solves a collection of differential phase measurements for a single attitude estimate and then considers perturbations to the initial estimate at each measurement epoch to produce a time-varying batch solution to the data. Although this is a widely used algorithm, there are certain disadvantages. First, an a priori attitude estimate must be given. Second, the algorithm is an iterative batch estimator that may produce erroneous estimates, depending on the accuracy of the a priori attitude estimate. Finally, if a large number of samples in the data collection are required to observe the motion, large-order matrix inversions may be required. Another method[10] performs a minimization on three Euler-angle attitude parameters independent of each other, followed by determining the integers. This approach has been shown to provide better convergence than Cohen's method and works well for noncoplanar baselines; however, singular conditions

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can exist at various attitude rotations and a significant amount of vehicle motion may be necessary for a solution.

In this paper, a new motion-based algorithm is derived. The main advantages of the new algorithm over the prior methods include the following: 1) it resolves the integer ambiguities without any a priori attitude knowledge, 2) it requires less computational effort because large matrix inverses are not needed, and 3) it is noniterative. The only disadvantage of the new algorithm is that it requires at least three noncoplanar baselines. The algorithm is first shown as a batch solution and then shown as a sequential solution. A covariance expression is also derived that can be used to bound the integer solution so that a sufficient integrity check for convergence can be developed. This is extremely useful in the sequential formulation because the solution can be found as the motion occurs, rather than taking a batch solution at a specific data collection interval. For these reasons, the new algorithm provides an attractive method for real-time ambiguity resolution.

This paper is organized as follows. First, the concept of the GPS phase difference measurement is introduced. Next, the new motion-based algorithm is derived. The representation of the GPS sightline vector in the body frame is reviewed, and the batch solution used to resolve the integer ambiguities is derived, followed by the sequential solution. Finally, the new algorithm is validated by using an actual GPS receiver with a hybrid dynamic simulator to simulate the vehicle motions of a low-altitude Earth-orbiting spacecraft.

GPS Sensor Model

In this section a brief background of the GPS phase difference measurement is shown. The main measurement used for attitude determination is the phase difference of the GPS signal received from two antennas separated by a baseline. The wave front angle and wavelength are used to develop a phase difference, as shown in Fig. 1. The phase difference measurement is obtained by[9]

$$b_l \cos\theta = \lambda(\Delta\phi - n) \tag{1}$$

where b_l is the baseline length (in centimeters), θ is the angle between the baseline and the line of sight to the GPS spacecraft, n is the integer part of the phase difference between two antennas, $\Delta\phi$ is the fractional phase difference (in cycles), and λ is the wavelength (in centimeters) of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, nonmilitary applications generally use the L1 frequency. The measured fractional phase difference can be expressed by

$$\Delta\phi = \boldsymbol{b}^T A\boldsymbol{s} + n \tag{2}$$

where $\boldsymbol{s} \in R^3$ is the normalized line-of-sight vector to the GPS spacecraft in a reference frame, $\boldsymbol{b} \in R^3$ is the baseline vector (in wavelengths), which is the relative position vector from one antenna to another, and $A \in R^{3\times3}$ is the attitude matrix, an orthogonal matrix with determinant 1 (i.e., $A^T A = I_{3\times3}$) representing the transformation between the two frames. The measurement model is given by

$$\Delta\tilde{\phi}_{ij} = \boldsymbol{b}_i^T A\boldsymbol{s}_j + n_{ij} + w_{ij} \tag{3}$$

where $\Delta\tilde{\phi}_{ij}$ denotes the phase difference measurement for the ith baseline and jth sightline, and w_{ij} represents a zero-mean Gaussian measurement error with standard deviation ϖ_{ij}, which is $0.5\,\text{cm}/\lambda = 0.026$ wavelengths for typical phase noise.[9]

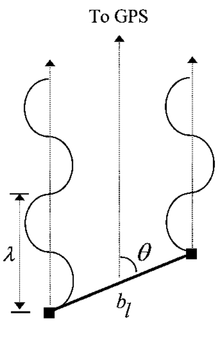

Fig. 1 GPS wavelength and wavefront angle.

Integer Ambiguity Resolution

In this section a new attitude-independent algorithm to resolve the integer ambiguities is presented. The problem is first converted into a form similar to the magnetometer-bias problem.[11,12] A batch solution for this problem is shown, followed by a sequential approach.

The new algorithm begins by representing the jth sightline vector in the body frame, $A\boldsymbol{s}_j$, as the sum of two components. The first component $\hat{\boldsymbol{s}}_j$ is a function of the measured fractional phase measurements, and the second $\boldsymbol{c}_j$ depends on the unknown integer phase differences. This representation is accomplished by minimizing the following loss function[13]:

$$J(A\boldsymbol{s}_j) = \frac{1}{2}\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\left(\Delta\tilde{\phi}_{ij} - n_{ij} - \boldsymbol{b}_i^T A\boldsymbol{s}_j\right)^2 \quad \text{for} \quad j = 1, 2, \ldots, N \tag{4}$$

where M is the number of baselines and N is the number of available sightlines. If at least three noncoplanar baselines exist, the minimization of Eq. (4) is straightforward and leads to

$$A\boldsymbol{s}_j = \hat{\boldsymbol{s}}_j - \boldsymbol{c}_j \tag{5a}$$

$$\hat{\boldsymbol{s}}_j = B_j^{-1}\left[\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\Delta\tilde{\phi}_{ij}\boldsymbol{b}_i\right] \tag{5b}$$

$$\boldsymbol{c}_j = B_j^{-1}\left[\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}n_{ij}\boldsymbol{b}_i\right] \tag{5c}$$

$$B_j = \sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\boldsymbol{b}_i\boldsymbol{b}_i^T \tag{5d}$$

Because the measurements are not perfect, Eq. (5a) is replaced by the effective measurement model

$$\hat{\boldsymbol{s}}_j = A\boldsymbol{s}_j + \boldsymbol{c}_j + \varepsilon_j \tag{6}$$

where $\boldsymbol{c}_j$ is a constant bias because the baselines are assumed constant, and ε_j is a zero-mean Gaussian process with covariance $R_j = B_j^{-1}$. This model is used for the actual attitude determination,[13] which we will not consider further in this paper.

The next step is to use an attitude-independent method to find the phase-bias vector $\boldsymbol{c}_j$ for each sightline, which gives all of the sightlines in both the body frame and the reference frame. The explicit integer phases are not needed for this solution, but it is important to check that they are close to integer values, as mentioned in the Introduction. In the general case the explicit integer phases can be found from the attitude solution. The three-baseline case ($M = 3$) is simpler, for in this case Eq. (5c) can be inverted to give

$$n_{ij} = \boldsymbol{b}_i^T\boldsymbol{c}_j \tag{7}$$

With more than three baselines, however, Eq. (5c) does not have a unique solution for $\boldsymbol{c}_j$, and so the M integer phases for sightline $\boldsymbol{s}_j$ cannot be found from $\boldsymbol{c}_j$ alone. We will consider the three-baseline case, which is the most common in practice. If more baselines are available, we are always free to select a three-baseline subset. Then, after the integer phases have been determined, a refined attitude estimate can be computed using all baselines (i.e., three baselines are sufficient to determine an attitude, which then may be used to resolve the integers corresponding to the other baselines).

To eliminate the dependence on the attitude, the orthogonality of A and Eq. (6) are used to give

$$\begin{aligned}\|\boldsymbol{s}_j\|^2 &= \|A\boldsymbol{s}_j\|^2 = \|\hat{\boldsymbol{s}}_j - \boldsymbol{c}_j - \varepsilon_j\|^2 \\ &= \|\hat{\boldsymbol{s}}_j\|^2 - 2\hat{\boldsymbol{s}}_j\cdot\boldsymbol{c}_j + \|\boldsymbol{c}_j\|^2 - 2(\hat{\boldsymbol{s}}_j - \boldsymbol{c}_j)\cdot\varepsilon_j + \|\varepsilon_j\|^2\end{aligned} \tag{8}$$

Next, following Alonso and Shuster,[12] the following effective measurement and noise are defined:

$$z_j \equiv \|\hat{s}_j\|^2 - \|s_j\|^2 \tag{9a}$$

$$v_j \equiv 2(\hat{s}_j - c_j)\cdot \varepsilon_j - \|\varepsilon_j\|^2 \tag{9b}$$

Then the effective measurement model is

$$z_j = 2\,\hat{s}_j \cdot c_j - \|c_j\|^2 + v_j \tag{10}$$

where v_j is approximately Gaussian for small ε_j with mean and variance given by, respectively,

$$\mu_j \equiv E\{v_j\} = -\mathrm{trace}\{R_j\} \tag{11}$$

and

$$\sigma_j^2 \equiv E\left\{v_j^2\right\} - \mu_j^2 = 4(\hat{s}_j - c_j)^T R_j(\hat{s}_j - c_j) - \mu_j^2 \tag{12}$$

Equations (9–12) define an attitude-independent algorithm because they do not contain the attitude matrix A.

The negative-log-likelihood function for the bias is given by

$$J(c_j) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[z_j(k) - 2\,\hat{s}_j(k)\cdot c_j + \|c_j\|^2 - \mu_j(k)\right]^2 + \log\sigma_j^2(k) + \log 2\pi\right\} \tag{13}$$

where L is the total number of measurement epochs, and the symbol k denotes the variable at time t_k. The maximum-likelihood estimate for c_j, denoted by c_j^*, minimizes the negative-log-likelihood function and satisfies

$$\left.\frac{\partial J(c_j)}{\partial c_j}\right|_{c_j^*} = \mathbf{0} \tag{14}$$

The minimization of Eq. (13) is not straightforward because the likelihood function is quartic in c_j. A number of algorithms have been proposed for estimating the bias (see Ref. 12 for a survey). The simplest solution is obtained by scoring, which involves a Newton–Raphson iterative approach. Another approach avoids the minimization of a quartic loss function by using a centered estimate. A statistically correct centered estimate is also derived in Ref. 12. Furthermore, Alonso and Shuster show a complete solution of the statistically correct centered estimate that determines the exact maximum likelihood estimate c_j^*. This involves using the statistically correct centered estimate as an initial estimate and iterating on a correction term using a Gauss–Newton method. Although this extension to the statistically correct centered estimate can provide some improvements, this part is not deemed necessary for the GPS problem because the estimated quantity for n_{ij} is rounded to the nearest integer.

Batch Solution

In this section the statistically correct centered estimate algorithm (see Ref. 12 for details) and its application to the integer ambiguity problem are shown. First, the following weighted averages are defined:

$$\bar{z}_j \equiv \bar{\sigma}_j^2 \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} z_j(k), \qquad \bar{s}_j \equiv \bar{\sigma}_j^2 \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} \hat{s}_j(k)$$
$$\bar{v}_j \equiv \bar{\sigma}_j^2 \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} v_j(k), \qquad \bar{\mu}_j \equiv \bar{\sigma}_j^2 \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} \mu_j(k) \tag{15}$$

where

$$\frac{1}{\bar{\sigma}_j^2} \equiv \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} \tag{16}$$

Next, the following variables are defined:

$$\tilde{z}_j(k) \equiv z_j(k) - \bar{z}_j, \qquad \tilde{s}_j(k) \equiv \hat{s}_j(k) - \bar{s}_j$$
$$\tilde{v}_j(k) \equiv v_j(k) - \bar{v}_j, \qquad \tilde{\mu}_j(k) \equiv \mu_j(k) - \bar{\mu}_j \tag{17}$$

The statistically correct centered estimate now minimizes the following loss function:

$$\tilde{J}(c_j) = \frac{1}{2}\sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)}[\tilde{z}_j(k) - 2\,\tilde{s}_j(k)\cdot c_j - \mu_j(k)]^2 \tag{18}$$

which is now a quadratic function in c_j. The minimization leads directly to

$$c_j^* = P_j \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)}[\tilde{z}_j(k) - \tilde{\mu}_j(k)]\,2\,\tilde{s}_j(k) \tag{19}$$

where the estimate error covariance is given by

$$P_j = \left[\sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} 4\,\tilde{s}_j(k)\tilde{s}_j^T(k)\right]^{-1} \tag{20}$$

The ambiguity for the ith baseline and jth sightline can be resolved by rounding the following to the nearest integer:

$$n_{ij} = b_i^T c_j^* \tag{21}$$

The integer error covariance, denoted by Q_{ij}, can be shown to be given by

$$Q_{ij} = b_i^T P_j b_i \tag{22}$$

Equation (22) can be used to develop an integrity check for the algorithm, using standard results on hypothesis testing.[14] For example, the procedure of rounding Eq. (21) to the nearest integer can be shown to have only a 0.0013 probability of selecting the wrong integer when $3\sqrt{Q_{ij}}$ is less than one-half.

Sequential Formulation

This section expands upon the batch solution so that a sequential estimate of the integers can be found. The main advantage of a sequential formulation is that the convergence (integrity) check can be made on the fly (i.e., in real time). The covariance in Eq. (20) is expanded to the $L+1$ time point, so that

$$P_j^{-1}(L+1) = \sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} 4\,\tilde{s}_j(k)\,\tilde{s}_j^T(k) + \frac{1}{\sigma_j^2(L+1)} 4\,\tilde{s}_j(L+1)\,\tilde{s}_j^T(L+1)$$
$$= P_j^{-1}(L) + \frac{1}{\sigma_j^2(L+1)} 4\,\tilde{s}_j(L+1)\,\tilde{s}_j^T(L+1) \tag{23}$$

From the matrix inversion lemma,[15] the following sequential formulation for the covariance is developed:

$$P_j(k+1) = K_j(k)P_j(k) \tag{24}$$

where

$$K_j(k) \equiv I - P_j(k)\,\tilde{s}_j(k+1) \times \left[\tilde{s}_j^T(k+1)P_j(k)\,\tilde{s}_j(k+1) + \tfrac{1}{4}\sigma_j^2(k+1)\right]\tilde{s}_j^T(k+1) \tag{25}$$

To derive sequential formulas for the quantities in Eq. (15), first consider the following identity:

$$\sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)} z_j(k) = \frac{1}{\bar{\sigma}_j^2(L)}\bar{z}_j(L) = \left[\sum_{k=1}^{L}\frac{1}{\sigma_j^2(k)}\right]\bar{z}_j(L) \tag{26}$$

Expanding out this expression using $L-1$ points in the summation yields

$$\frac{1}{\bar{\sigma}_j^2(L-1)}\bar{z}_j(L-1)+\frac{1}{\sigma_j^2(L)}z_j(L) = \left[\frac{1}{\bar{\sigma}_j^2(L-1)}+\frac{1}{\sigma_j^2(L)}\right]\bar{z}_j(L) \tag{27}$$

and so

$$\bar{z}_j(L)=\frac{\sigma_j^2(L)\bar{z}_j(L-1)+\bar{\sigma}_j^2(L-1)\,z_j(L)}{\sigma_j^2(L)+\bar{\sigma}_j^2(L-1)} \tag{28}$$

Therefore, the following sequential expressions for the quantities in Eq. (15) are given:

$$\bar{z}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)} \times\left[\sigma_j^2(k+1)\bar{z}_j(k)+\bar{\sigma}_j^2(k)z_j(k+1)\right] \tag{29a}$$

$$\bar{\boldsymbol{s}}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)} \times\left[\sigma_j^2(k+1)\,\bar{\boldsymbol{s}}_j(k)+\bar{\sigma}_j^2(k)\,\hat{\boldsymbol{s}}_j(k+1)\right] \tag{29b}$$

$$\bar{\mu}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)} \times\left[\sigma_j^2(k+1)\bar{\mu}_j(k)+\bar{\sigma}_j^2(k)\mu_j(k+1)\right] \tag{29c}$$

where

$$\frac{1}{\bar{\sigma}_j^2(k+1)}=\frac{1}{\bar{\sigma}_j^2(k)}+\frac{1}{\sigma_j^2(k+1)} \tag{30}$$

The estimated bias in Eq. (19) can also be found in a similar manner, so that

$$\boldsymbol{c}_j^*(k+1)=K_j(k)\,\boldsymbol{c}_j^*(k)+\frac{1}{\sigma_j^2(k+1)} \times[\tilde{z}_j(k+1)-\tilde{\mu}_j(k+1)]\,2P_j(k+1)\,\tilde{\boldsymbol{s}}_j(k+1) \tag{31}$$

Because the baselines are constant, Eqs. (21) and (22) can be used directly to determine the sequential integer value and error covariance, given by

$$n_{ij}(k)=\boldsymbol{b}_i^T\boldsymbol{c}_j^*(k) \tag{32a}$$

$$Q_{ij}(k)=\boldsymbol{b}_i^T P_j(k)\boldsymbol{b}_i \tag{32b}$$

The complete solution proceeds as follows. First, use Eqs. (5b) and (5d) to represent the sightline vectors in the body frame. Then, perform an initial batch solution using Eqs. (15–20) to initialize the sequential routine (an accurate initial estimate is not required as will be seen in the results section). Then, perform a sequential estimate for the integers using Eqs. (24), (25), and (29–32). Finally, continue until the covariance in Eq. (32b) is below a prespecified value.

There are many advantages of the new algorithm. First, the algorithm is fully autonomous (i.e., it requires no a priori information such as an a priori attitude guess). Second, the largest matrix inverse is of a 3×3 matrix, which makes the algorithm computationally efficient and stable. Third, it is noniterative, which makes it suitable for a sequential formulation. This has a significant advantage because the convergence can be checked during the actual motion in the vehicle. Finally, the integers for other sightlines can be easily resolved by calling the same subroutine. Therefore, the algorithm can be implemented easily using all available sightlines, and attitude determination can begin once the integers corresponding to two sightlines have been resolved. For these reasons the new algorithm provides an attractive approach to resolve the integers.

Hardware Simulation and Results

A hardware simulation of a typical spacecraft attitude determination application was undertaken to demonstrate the performance of the new algorithm. For this simulation a Northern Telecom 40-channel, 4-rf-output STR 2760 unit was used to generate the GPS signals that would be received at a user-specified location and velocity. The signals are then provided directly (i.e., they are not actually radiated) to a GPS receiver that has been equipped with software tracking algorithms that allow it to operate in space (see Fig. 2).

The receiver that was used was a Trimble TANS Vector, which is a six-channel, four-rf-input multiplexing receiver that performs three-axis attitude determination using GPS carrier phase and line-of-sight measurements. This receiver software was modified at Stanford University and NASA Goddard Space Flight Center to allow it to operate in space. This receiver model has been flown and operated successfully on several spacecraft, including REX-II, OAST-Flyer, GANE, Orbcomm, Microlab, and others.

The simulated motion profile was for an actual spacecraft, the Small Satellite Technology Initiative (SSTI) Lewis satellite, which carried an experiment to assess the performance of GPS attitude determination on orbit. Although the spacecraft was lost due to a malfunction not related to the GPS experiment shortly after launch, this motion profile is nonetheless very representative of the types of attitude determination applications. The orbit parameters and pointing profile used for the simulation are given in Table 1.

The simulated SSTI Lewis spacecraft has four GPS antennas that form three baselines. The antenna separation distances are 0.61, 1.12, and 1.07 m, respectively. One antenna (in baseline 3) is located 0.23 m out of plane (below) the other three antennas. On the spacecraft the antennas are mounted on pedestals with ground planes to minimize signal reflections and multipath. For the simulation multipath errors are introduced using a simple Markov process with a time constant of 5 min and a standard deviation of 0.026 wavelengths.[16] The baseline vectors in wavelengths are given by

$$\boldsymbol{b}_1=\begin{bmatrix}2.75\\1.64\\-0.12\end{bmatrix},\qquad \boldsymbol{b}_2=\begin{bmatrix}0.00\\6.28\\-0.17\end{bmatrix},\qquad \boldsymbol{b}_3=\begin{bmatrix}-3.932\\3.93\\-1.23\end{bmatrix} \tag{33}$$

Line biases are first determined before the new algorithm is tested to resolve the integer ambiguities. The GPS raw measurements are

Table 1 SSTI Lewis orbit parameters

Parameter	Value
Semimajor axis a, km	6901.137
Inclination i, deg	97.45
Right ascension of ascending node, deg	−157.1
Eccentricity e	0.0001
Pointing profile	Earth pointed
Launch date	Aug. 22, 1997

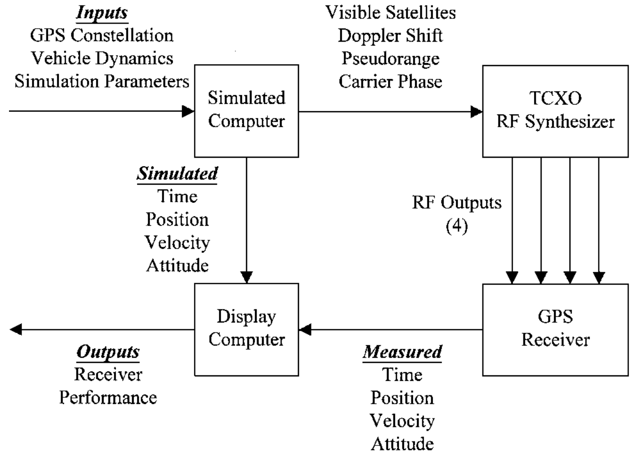

Fig. 2 Hardware simulation block diagram.

processed at 1 Hz over a 40-min simulation. During the simulated run, a minimum of three visible GPS satellites are in sight at all times. However, resolution of the ambiguous integers for the phase measurements from any specific GPS satellite requires that it remain in view continuously until the sequential algorithm converges. In practice all available sightlines should be processed because attitude determination requires the integers to be resolved for two GPS satellites simultaneously. The simulation contains a number of 8-min spans when sightlines to two specific GPS satellites are continuously available for the ambiguity resolution algorithm.

As mentioned previously, the first step in the algorithm involves using the baselines and phase difference measurements to convert the sightline vector into the body frame, using Eqs. (5b) and (5d). Then a small batch run is used to initialize the sequential routine. For this case, 1 min of data was used to perform the initialization (simulation results indicate that convergence is possible using only 10 s of data). Again, only two sightlines are required to determine the attitude. Computed solutions for the first sightline are shown in Fig. 3 (actual integer values are found by rounding the computed values to the nearest integer). For this case the integers have been resolved within 2 min. A plot of the $3\sqrt{Q_{ij}}$ values is shown in Fig. 4 for the first sightline (a suitable integrity check is given when $3\sqrt{Q_{ij}}$ is below one-half). Clearly, the integrity check shows that the ambiguities are resolved within 5 min. Note that this is a sufficiency test (i.e., the integers may be resolved well before 5 min, which is seen in this case). Computed solutions for the second sightline are shown in Fig. 5. For this case, all of the ambiguities have been resolved within 5 min. The sharp jump just after 4 min is due to a rapid change in the sequential variance in Eq. (12). This jump is also present in Fig. 3, but it is not as pronounced as in Fig. 5

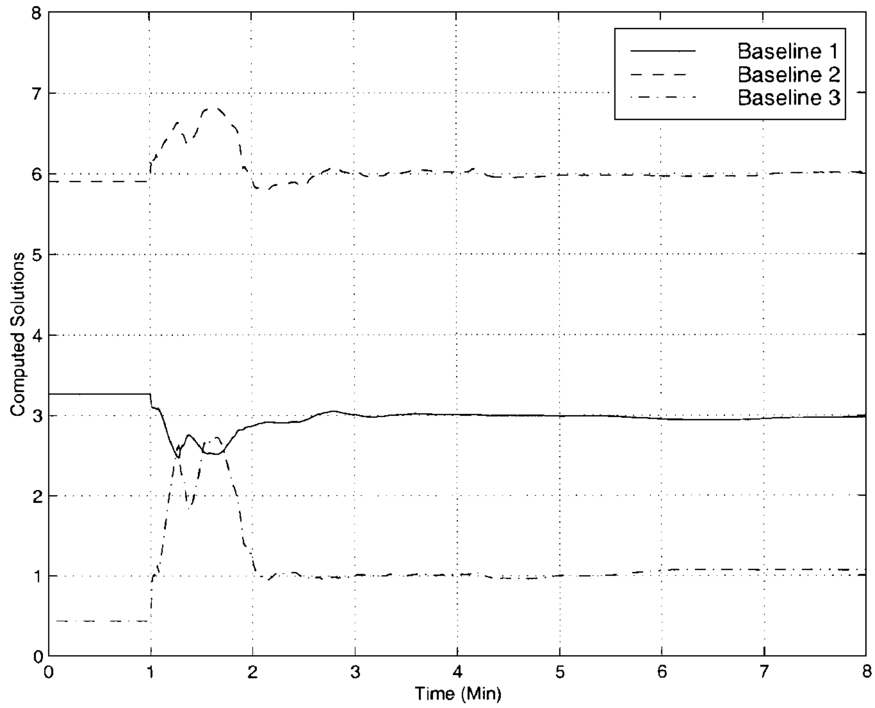

Fig. 3 Computed values for first sightline.

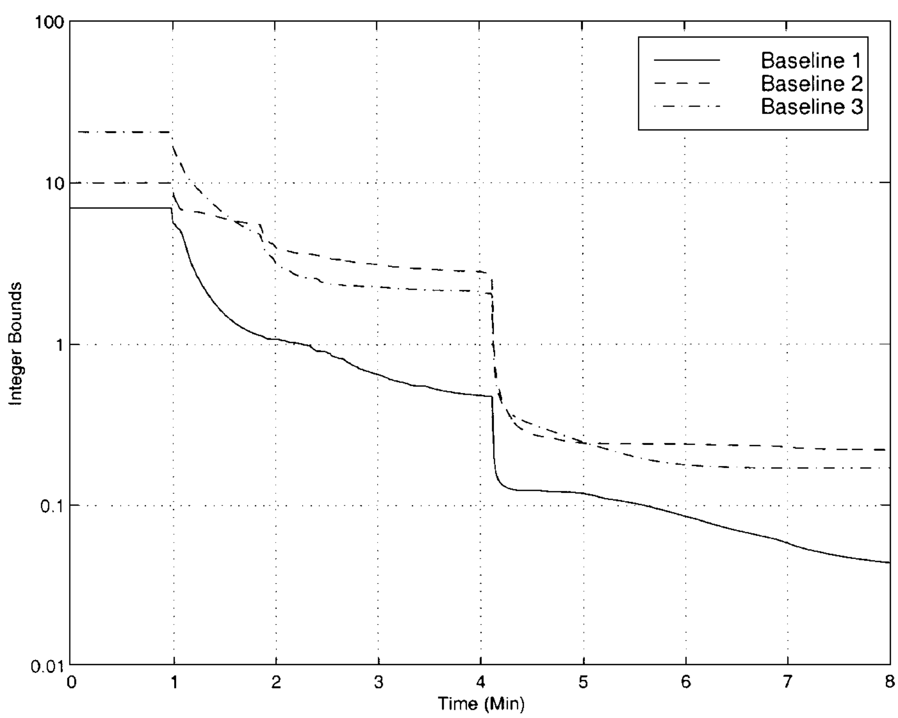

Fig. 4 Integrity check for first sightline.

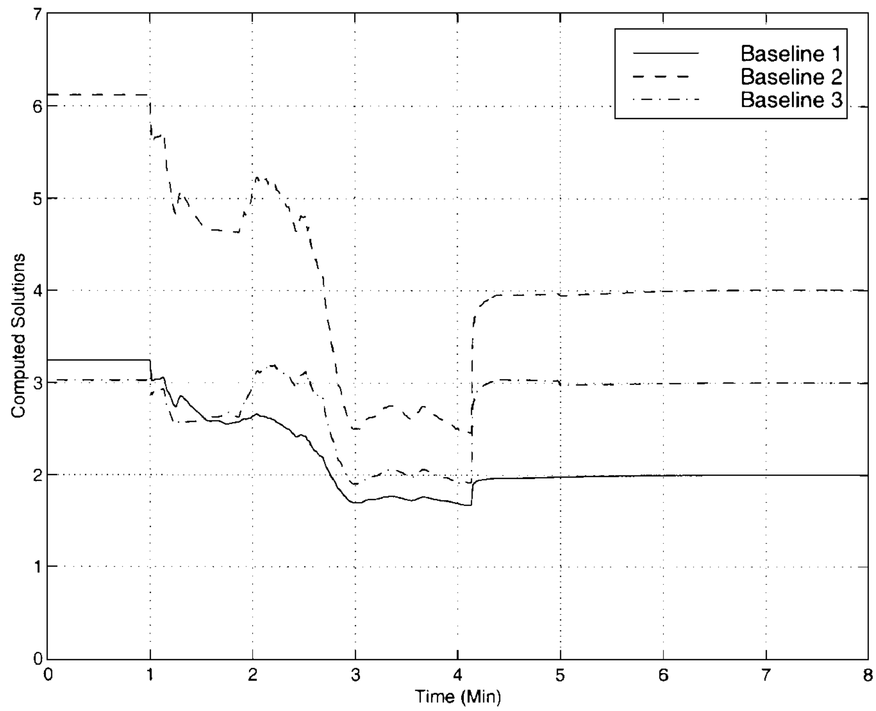

Fig. 5 Computed values for second sightline.

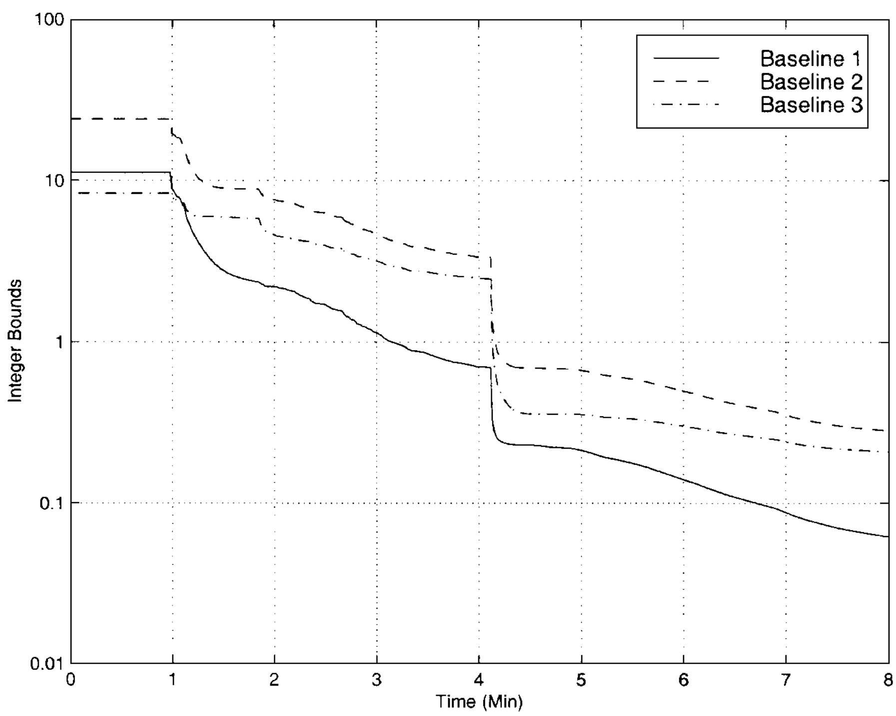

Fig. 6 Integrity check for second sightline.

because $\mathbf{c}_j$ has converged before this time. A plot of the integrity check for the second sightline is shown in Fig. 6. The integrity check shows that the ambiguities are resolved within 7 min. This hardware simulation of a spacecraft clearly demonstrates that the new algorithm presented in this paper provides an accurate method to resolve the integer ambiguities with even slight vehicle motion.

Conclusions

In this paper a new algorithm was developed for GPS integer ambiguity resolution. The new algorithm has several advantages over previously existing algorithms. First, the algorithm is attitude independent so that no a priori attitude estimate (or assumed vehicle motion) is required. Second, the algorithm is sequential so that it may be implemented in real time. Also, a suitable integrity check can be used to determine when the determined values have converged to the correct values. Finally, the algorithm is computationally efficient because only a 3×3 matrix inverse is required and the same subroutine can be used on different sightlines. The only disadvantage of the new algorithm is that it requires at least three noncoplanar baselines. The algorithm was tested using a GPS hardware simulator to simulate the motions of a typical low-altitude Earth-orbiting spacecraft. Results indicated that the new algorithm provides a viable and attractive means to resolve the integer ambiguities effectively.

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